

MARKER CODE				



STUDENT ENROLMENT NUMBER (SEN)									

TONGA NATIONAL FORM SEVEN CERTIFICATE

2024

MATHEMATICS WITH CALCULUS

QUESTION AND ANSWER BOOKLET

Time allowed: 3 Hours

INSTRUCTIONS:

1. Write your **Student Enrolment Number (SEN)** on the top right-hand corner of this page.
2. This paper consists of **FOUR SECTIONS** and is out of 70 weighted scores.

SECTION	STRANDS	TOTAL SKILL LEVEL
ONE	NUMBERS	9
TWO	ALGEBRA	14
THREE	DIFFERENTIATION	23
FOUR	INTEGRATION	24
TOTAL		70

3. Answer ALL QUESTIONS. Write your answers in the spaces provided in this booklet.
4. Use a **BLUE** or **BLACK** ball point pen only for writing. Use a pencil for drawing if required.
5. If you need more space for answers, ask the supervisor for extra paper. Write your **Student Enrolment Number (SEN)** on each additional sheet, number the questions clearly and insert them in the appropriate places in this booklet.
NOTE: A **4-page booklet (No.504/2)** containing mathematical formulae and tables will be provided.
6. Check that this booklet contain pages **2-19** in the correct order and that page 17-19 has been deliberately left blank.

YOU MUST HAND IN THIS BOOKLET TO THE SUPERVISOR BEFORE YOU LEAVE THE EXAMINATION ROOM.

SECTION ONE:

NUMBERS

a. Let $u = 3 + i$ and $v = -2 - 2i$. Find:

i. $\bar{v}$

Skill level 1	
1	
0	
NR	

ii. $\frac{u}{\bar{v}}$

Skill level 2	
2	
1	
0	
NR	

iii. Calculate the argument of v .

Skill level 2	
2	
1	
0	
NR	

b. Simplify $\sqrt{2}(\sqrt{5} - \sqrt{3})$.

Skill level 1	
1	
0	
NR	

c. Simplify $\frac{2^5 \cdot 2^3}{2^7}$.

Skill level 1	
1	
0	
NR	

d. Use the properties of logarithms to simplify $\log_2 16 + \log_2 \frac{1}{4} - \log_2 2$.

Skill level 2	
2	
1	
0	
NR	

SECTION TWO:

ALGEBRA

a. Factorise $x^2 + 4$.

Skill level 2	
2	
1	
0	
NR	

b. Simplify $\frac{16^{2n}}{8^{4n}}$.

Skill level 2	
2	
1	
0	
NR	

c. Let $z = \text{cis } \theta$. Prove that $z + \frac{1}{z} = 2\cos\theta$.

Skill level 3	
3	
2	
1	
0	
NR	

d. Solve $\sqrt{x-2} = x-1$.

Skill level 3	
3	
2	
1	
0	
NR	

e. Determine the solutions for the equation $z^3 = -8$. Give your answers in polar form $r\text{cis}\theta$, where angles are in radians.

Skill level 4	
4	
3	
2	
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SECTION THREE:

DIFFERENTIATION

- a. Find the derivative of the following.
You do not need to simplify your answer.

i. $f(x) = e^{2x}$

Skill level 1	
1	
0	
NR	

ii. $f(x) = x^2e^{x^2}$

Skill level 2	
2	
1	
0	
NR	

- b. Consider the curve given by the function $f(x) = x^3 - 3x^2 + 2x$. Determine the equation of the tangent line to the curve at the point where $x = 1$.

Skill level 3	
3	
2	
1	
0	
NR	

- c. Calculate the stationary points of $f(x) = \frac{x^2}{1-x}$, and determine the nature of f at such points.

Skill level 3	
3	
2	
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- d. Describe the condition where $\lim_{x \rightarrow a} f(x)$ exists.

Skill level 2	
2	
1	
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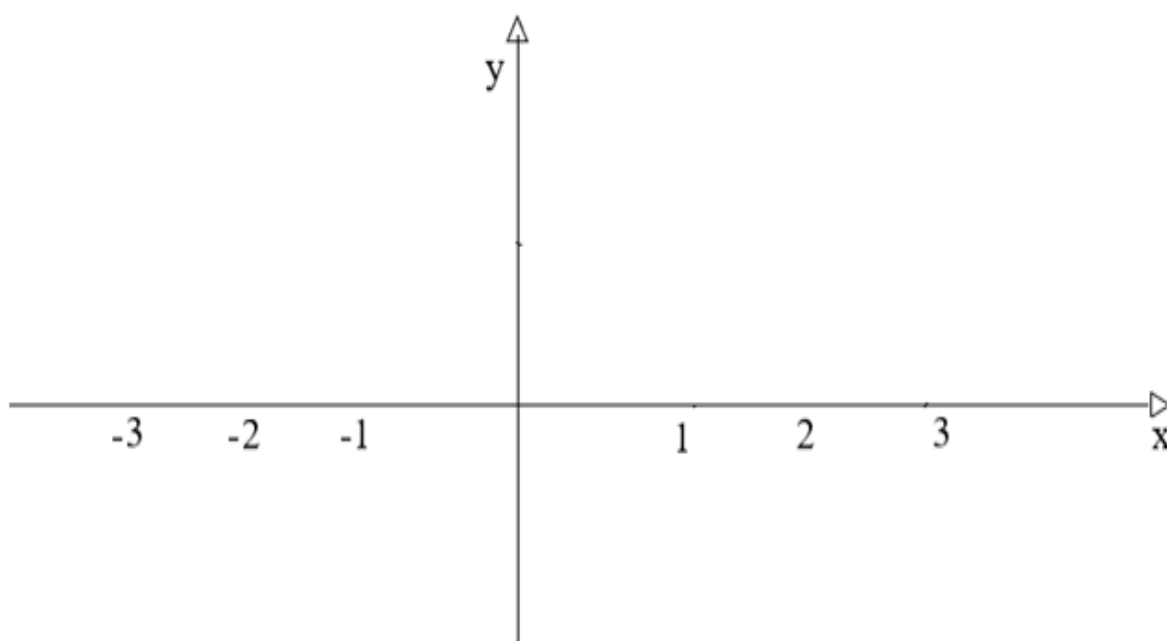
- e. The first derivative of the parametric equations $x = \theta - \sin \theta$ and $y = 1 - \cos \theta$ is $\frac{dy}{dx} = \frac{\sin \theta}{1 - \cos \theta}$. Find the second derivative $\frac{d^2y}{dx^2}$ in terms of θ .

Skill level 2	
2	
1	
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- f. Use implicit differentiation to find $\frac{dy}{dx}$ if $x^2 + y^2 = \sin(xy)$.

Skill level 3	
3	
2	
1	
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- g. Sketch a graph of $f(x)$ on the interval $-3 < x < 3$ that satisfies the following conditions:
- Differentiable at $0 < x < 3$ and $-3 < x < -1$
 - Discontinuous at $x = 0$
 - Increasing on $-3 < x < -1$
 - Decreasing on $-1 < x < 3$
 - Stationary Point at $x = 2$
 - Continuous but not differentiable at $x = -1$



Skill level 3	
3	
2	
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- h. An ice sculpture in the form of a sphere melts in such a way that it maintains its spherical shape. The volume of the sphere is decreasing at a constant rate of 2π cubic meters per hour. At what rate in square meters per hour, is the surface area of the sphere decreasing at the moment when the radius is 5 meters?
 (Note: For a sphere of radius r , the surface area is $4\pi r^2$ and the volume is $\frac{4}{3}\pi r^3$.)

Skill level 4	
4	
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SECTION FOUR

INTEGRATION

- a. Find the integrals of the following. **You do not need to simplify your answer.**
- i.

Skill level 1	
1	
0	
NR	

ii. $\int (\sin 3x) dx$

Skill level 1	
1	
0	
NR	

iii. $\int \left(\frac{1}{2x}\right) dx$

Skill level 1	
1	
0	
NR	

∫

b. Evaluate the definite integral $\int_{-1}^1 (x - 1)^2 dx$.

Skill level 2	
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Skill level 3	
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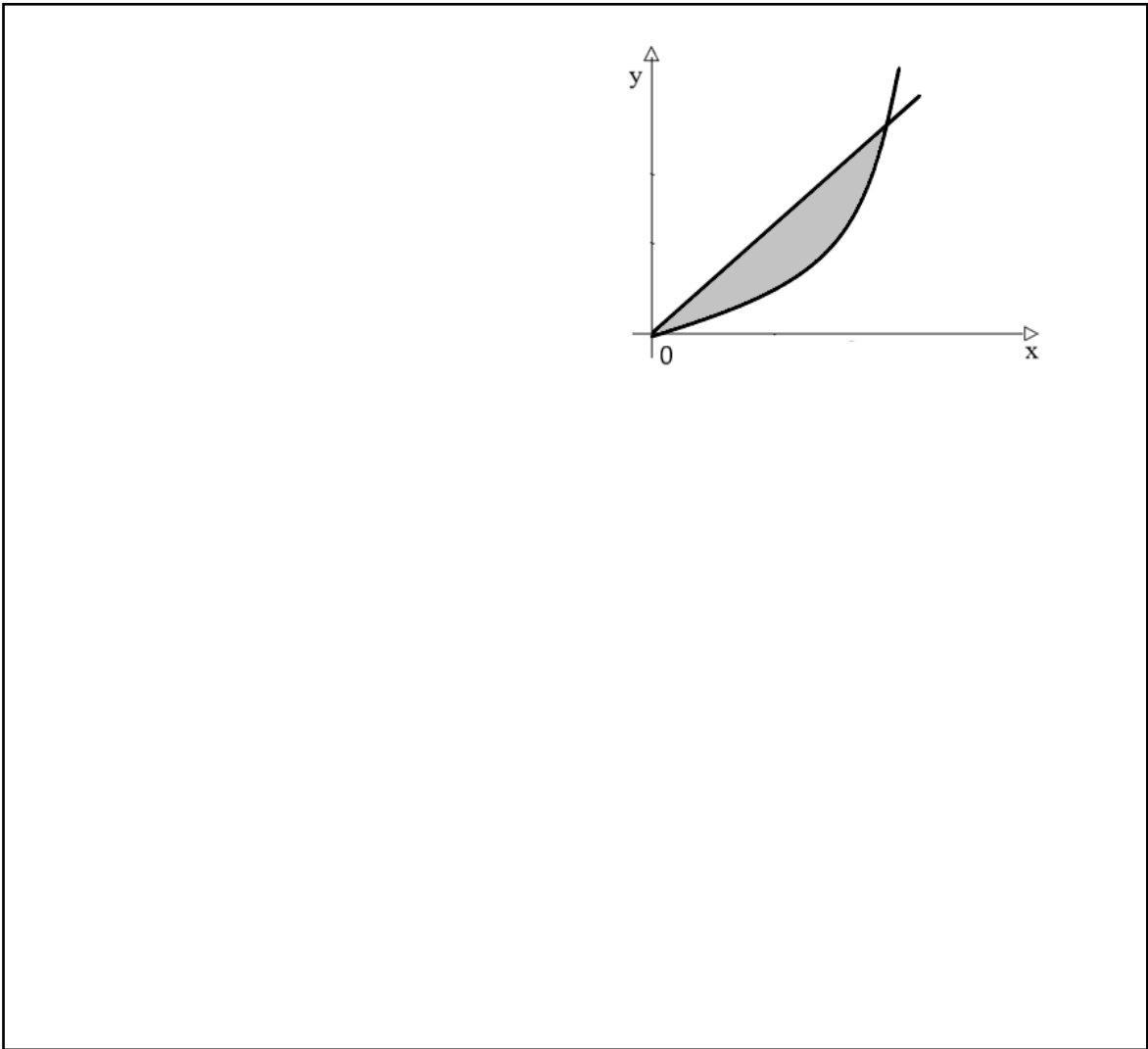
- c. Using a suitable trigonometric identity, find $\int \sin 2x \cos 2x \, dx$
- d. A curve is given by the following coordinates:

x	1	2	3	4	5	6	7	8	9
y	0	0.6	0.7	1.2	1.4	1.5	1.7	1.8	2.0

Correct to 1 dp, use Simpson's Rule to find the area under the curve represented by the above table.

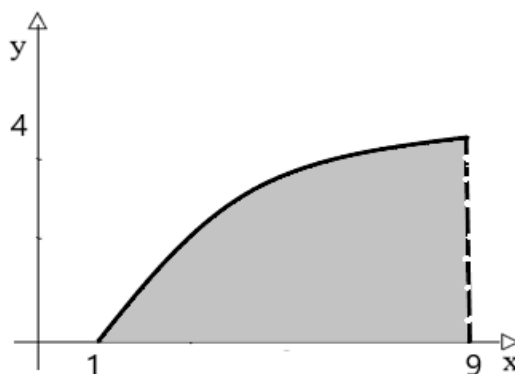
Skill level 2	
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- e. Find the shaded area between the curve $y = x^3$ and $y = x$ as shown in the diagram below.



Skill level 3	
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- f. The shaded area in the diagram lies between the curve $y = 4 - \frac{4}{x}$, the x-axis and the lines $x = 1$ and $x = 9$. Find the volume of the solid of revolution when the shaded area revolves around the x-axis.



Skill level 3	
3	
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0	
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g. Solve the differential equation $\frac{dy}{dx} = y^2 e^{-x}$ if $y = 1$ when $x = 0$.

Skill level 4	
4	
3	
2	
1	
0	
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h. Evaluate the definite integral $\int_{\frac{\pi}{2}}^{\pi} \sin^2 x dx$.

Skill level 4	
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